21 The Snow Elf's Christmastime Quest

Author: Prof. Dr. Daniel Gembris (Duale Hochschule Sachsen in Dresden)

Challenge

The diligent elf Willi has the important task of clearing the sports field of snow for the big Christmas party of the elves. He proceeds systematically: he divides the sports field into equally sized squares and clears one square completely before starting the next. The rest of the sports field, which cannot be divided into squares, he cleans afterwards. But it doesn't stop there - he wants to use the snow to create magnificent snowballs! Thanks to Willi's years of experience in forming snowballs, each of his snowballs turns out perfectly and has an exact diameter of 1.5 m.

The following information is given:

Information about the snow

- The sports field is equally covered with 5 cm of fresh snow.
- The snow has a density of $50\,\mathrm{kg/m^3}$.
- Forming the snow into a snowball compresses it, increasing its density to $100\,\mathrm{kg/m^3}$.

Willi's snow clearing strategy for one square

- **Division of the area and choice of lane width**

  The square with side length a is divided into parallel clearing lanes, all of which are parallel to one side of the square. The width b of a lane corresponds to the width of the snow shovel. This width is chosen such that Willi pushes at most 5 kg of snow at a time while clearing a lane.

Willi therefore calculated the lane width according to his maximum pushing force.

The lanes lie directly next to each other and together cover the entire area of the square. If the side length a is not an integer multiple of b, the last lane is narrower. In this case, it must be ensured that, when clearing this narrower lane, the snow of the surrounding squares remains untouched.

- **Movement while clearing a lane**

  Willi starts at one corner of the square. To position himself to clear the first lane he moves along the edge of the square by a distance of $\frac{b}{2}$, so that the snow shovel lies centered on the first lane. From this position, he clears the lane by pushing the snow of that lane in a straight line over the entire length a in the pushing direction up to the edge of the square.

  While clearing, the snow completely remains within the current lane. No snow is left behind and none ends up on neighboring lanes or squares. In the following considerations, the snow pushed to the edge is assumed to lie along the corresponding side of the square.

- **Change between to lanes**

  After a lane has been completely cleared, Willi first walks back along the same lane to the starting side of the square. He then moves perpendicular to the pushing direction by exactly one lane width b and positions himself at the beginning of the next lane. There, he repeats the clearing process following the same pattern.

  This procedure is continued lane by lane until all lanes of the square have been cleared. If the last lane is narrower, the lateral offset is adjusted according to its width.

- **Snow collection point and end of snow pushing**

  The snow collection point of a square is the corner that lies diagonally opposite the starting corner. After clearing the last lane, Willi does not walk back. Instead, he moves directly to the collection point. (Movements along the edge of the square are carried out independently of the snow located there.)

  There, Willi replaces the snow shovel with a bucket. The snow that has been pushed together along the edge of the square is then transported away and carried back to the snow collection point.

Information about the snow-transport

- Generally, Willi carries the snow in 5-kg portions; only during the final transport may the amount of snow collected be smaller.
- In order not to walk unnecessarily far before he has collected 5 kg of snow, he positions himself along the side of the square so that he can collect snow from both sides. Therefore, he moves along the side of the square that he pushed the snow to until he has passed 2.5 kg of snow (or has reached the end of the square). Willi then scoops 2.5 kg of snow from each side of his position, or whatever remains at the end of the square, and carries it to the snow collection point.

Willi's speed and time assumptions

- He moves at a constant speed of 1 m/s - both with and without a bucket or snow shovel.
- The time required to put down and pick up the bucket and shovel, to fill and empty the bucket, as well as to form the snowballs, is not taken into account.

Your Challenge

1. **Determine the side length** a of the square that is needed for a snowball.
2. **Determine the width** b **of the snow shovel** that Willi uses.
3. **Calculate the time** $t_{\mathbf{total}}$ **Willi needs** to transport the snow from a single square to the respective snow collection point, i.e. the time to push the snow with the snow shovel as well as the time for the transport with the bucket. The activity is complete when Willi ends up standing at the collection point.

Give all results in the following units and with the following rounding:

- a in meters, rounded to one decimal place.
- b in meters, rounded to two decimal places.
- t_{total} in minutes, rounded to one decimal place.

Possible Answers:

1. $a = 8.4\,\mathrm{m}$, $b = 0.22\,\mathrm{m}$, $t_{\mathrm{total}} = 10.1\,\mathrm{min}$
2. $a = 8.4\,\mathrm{m}$, $b = 0.22\,\mathrm{m}$, $t_{\mathrm{total}} = 13.4\,\mathrm{min}$
3. $a = 8.4\,\mathrm{m}$, $b = 0.22\,\mathrm{m}$, $t_{\mathrm{total}} = 15.2\,\mathrm{min}$
4. $a = 8.4\,\mathrm{m}$, $b = 0.24\,\mathrm{m}$, $t_{\mathrm{total}} = 10.1\,\mathrm{min}$
5. $a = 8.4\,\mathrm{m}$, $b = 0.24\,\mathrm{m}$, $t_{\mathrm{total}} = 15.2\,\mathrm{min}$
6. $a = 9.0\,\mathrm{m}$, $b = 0.22\,\mathrm{m}$, $t_{\mathrm{total}} = 10.1\,\mathrm{min}$
7. $a = 9.0\,\mathrm{m}$, $b = 0.22\,\mathrm{m}$, $t_{\mathrm{total}} = 13.4\,\mathrm{min}$
8. $a = 9.0\,\mathrm{m}$, $b = 0.24\,\mathrm{m}$, $t_{\mathrm{total}} = 10.1\,\mathrm{min}$
9. $a = 9.0\,\mathrm{m}$, $b = 0.24\,\mathrm{m}$, $t_{\mathrm{total}} = 13.4\,\mathrm{min}$
10. $a = 9.0\,\mathrm{m}$, $b = 0.24\,\mathrm{m}$, $t_{\mathrm{total}} = 15.2\,\mathrm{min}$

Project Reference:

Task from an external source. The author is a lecturer in mathematical and scientific fundamentals at the Duale Hochschule Sachsen in Dresden.