



12 Baking by Chance

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Challenge

In the Christmas bakery, as every year, preparations for the Christmas cookies begin on December 1, 2025. To ensure that enough cookies are ready on Christmas Eve, the elves decide to bake two sheets at a time.

Therefore, before each baking cycle, two baking sheets are prepared, one on the left side of the oven for the upper rack and one on the right side of the oven for the lower rack. Unfortunately, there are only two different cookie cutters: stars and Christmas trees. For the first baking cycle, the sheets are randomly¹ filled with cookies in these two shapes. Each baking sheet holds exactly 1000 cookies, regardless of the shape. Both baking sheets are then placed in the oven and the cookies are baked for 15 minutes at 175 degree Celsius.

Simultaneously, new sheets are prepared for the next baking cycle, one on the left and one on the right of the oven, each holding 1000 cookies again. The shape of each cookie on the left baking sheet is determined by copying the shape of a randomly selected cookie from the upper sheet in the oven. Similarly, for the right baking sheet, the shape of each new cookie is determined by the shape of a randomly selected cookie from the lower sheet in the oven.

In accordance with elf labor regulations, exactly 20 baking cycles are performed each day. The operation begins on December 1, 2025.

¹Throughout this problem, „randomly“ always means uniformly at random.

What is the probability (rounded to one decimal place) that two randomly chosen cookies finished in the final baking process on December 24, 2025, have the same shape if they are

- (a) from the same baking sheet, and
- (b) from different baking sheets?

Possible answers:

- 1. (a) 0,5 (b) 0,5
- 2. (a) 0,6 (b) 0,5
- 3. (a) 0,7 (b) 0,5
- 4. (a) 0,5 (b) 0,6
- 5. (a) 0,6 (b) 0,6
- 6. (a) 0,7 (b) 0,6
- 7. (a) 0,5 (b) 0,7
- 8. (a) 0,6 (b) 0,7
- 9. (a) 0,7 (b) 0,7
- 10. none of the above solutions

Project Reference:

Project EF4-7 dealt with various population models. The process described in this exercise is similar to a Wright-Fisher model, which was one of the first population models in mathematics.