9 Elves on the Elevator

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Challenge

It's Christmas Eve at the North Pole and Santa Claus is preparing for his grand journey. His sleigh, polished to a twinkling shine, waits atop the workshop tower, on the 10th floor. Down on the ground floor, the 0th floor, Santa stands in front of the elevator, rubbing his weary knees after a long night of toy inspections. As the doors open with a cheerful ding, he takes a first step towards them with a sigh of relief - at last, a lift bringing him to his sleigh instead of endless stairs and narrow chimneys!
But before he can step inside, a giggling swarm of eight mischievous elves darts past him. In a flurry of tiny hands and jingling laughter, they jab at the control panel, lighting up a merry constellation of buttons - every floor, even the 10th, fair game to their mischief. Each elf presses exactly one of the buttons $1, 2, \ldots, 10$ (uniformly at random) with the light-hearted whimsy typical of Christmas elves.
Santa groans and covers his eyes with a gloved hand, thinking, "I should have taken the stairs...". Before he can manage to look at the lit-up panel to see which buttons the elves have pushed, surrounded by his merry band of button-happy troublemakers, he asks himself, "What are the chances that, before reaching my sleigh on the 10th floor, the elevator will stop at exactly five different other floors along the way?

Remark: The elevator starts on the 0th floor and, going from bottom to top, makes exactly one stop at every floor whose button has been pushed at least once. As Santa enters the elevator, he presses the button for the 10th floor, where his sleigh is waiting.

Possible answers:

1. $\frac{6067920}{8^8} \approx 0.3617 \approx 36\%$.
2. $\frac{452307240}{8^{10}} \approx 0.4212 \approx 42\%$.
3. $\frac{17781120}{9^8} \approx 0.4131 \approx 41\%$.
4. $\frac{40007520}{10^8} \approx 0.4001 \approx 40\%$.
5. $\frac{44007520}{10^8} \approx 0.4401 \approx 44\%$.
6. $\frac{52007520}{10^8} \approx 0.5201 \approx 52\%$.
7. $\frac{52007520}{11^8} \approx 0.2426 \approx 24\%$.
8. $\frac{62007520}{11^8} \approx 0.2893 \approx 29\%$.
9. $\frac{64007520}{11^8} \approx 0.2986 \approx 30\%$.
10. $\frac{80015040}{11^8} \approx 0.3733 \approx 37\%$.