The Wrapping Room Riddle

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Challenge

In Santa Claus's wrapping room, the elves are rushing to finish wrapping the last presents in time for Christmas Eve. But there is a problem – **the wrapping-paper machine is broken!**

Instead of cutting all possible shapes of wrapping paper, the machine can only cut the paper into **five special shapes** (see Figure 1). Each shape consists of so-called identical unit elf squares (all small squares have exactly the same size), and the machine can **neither rotate nor flip** them – every piece must be used exactly as it is shown in Figure 1.

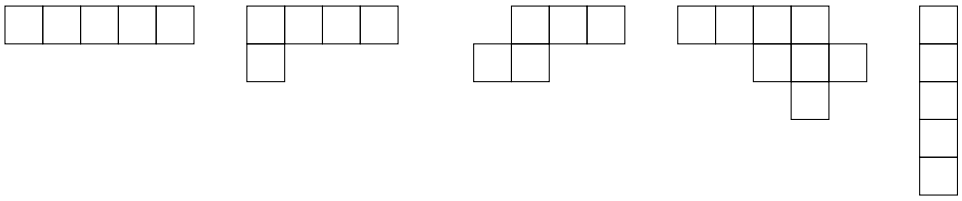

Figure 1: The five special shapes.

Fortunately, each of these pieces can still be used for wrapping certain presents:

- **Long strip** – used to wrap toy trains.
- **L-shape** – fits around boxes with corners.
- **Small step shape** – covers stacked books.

- **Water pistol** – for the water pistols in the gift boxes.
- **Straight column** – for candy canes.

The elves now need to cut some **rectangular pieces of size** $m \times n$ with the machine, where $m, n \geq 1$. Here, m is the number of rows and n is the number of columns of the rectangular piece.
They want to determine **for which pair** (m, n), listed in the possible answers below, the entire rectangle can be filled **using only these five shapes**, **with no leftover space**.
Can you help them figure it out before Santa leaves for his midnight journey?

Possible Answers

1. $m = 2021,\ n = 2024$
2. $m = 2021,\ n = 2029$
3. $m = 2022,\ n = 2026$
4. $m = 2022,\ n = 2028$
5. $m = 2023,\ n = 2024$
6. $m = 2023,\ n = 2027$
7. $m = 2024,\ n = 2026$
8. $m = 2024,\ n = 2027$
9. $m = 2024,\ n = 2029$
10. None of the other answers are true.