4 Cutting Christmas Cookies, Again!

Authors: Pim van 't Hof (University of Twente)
Project: 4TU.AMI

Challenge

Pixies Pi and Pie are at it again: just like last year, they are baking perfectly round Christmas cookies. For this reason, they prepared a large circular piece of dough yesterday and put it in the fridge overnight to chill. When they opened the fridge this morning, they discovered that a large circular piece of dough had been cut out. (They suspect pixie Pastry to have pinched that circular piece of dough to use it as a base for her annual Christmas pie.)

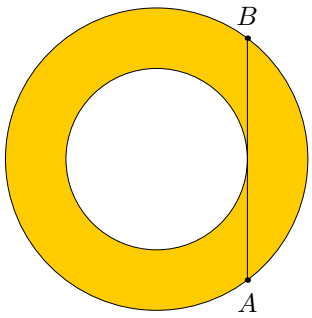

Figure 1: The piece of dough the pixies Pi and Pie found in the fridge.

Figure 1 shows the piece of dough that pixies Pi and Pie found in the fridge this morning. The boundary of the original piece of dough and the boundary of the large circular hole form concentric circles, i.e. they share the same center but have different radii. The line segment AB is a chord of the large circle that is tangent to the boundary of the circular hole. The length of AB is 36 cm.

Because the pixies Pi and Pie still want perfectly round Christmas cookies but only have two relatively large circular cookie cutters, they decide to knead the remaining piece of dough into a ball and roll it out into a circular piece of dough with exactly the same area as the piece of dough they found in the fridge this morning. From this circular piece of dough, they cut out three cookies: a large one and two smaller ones. Figure 2 shows the piece of dough that remains after the cookies have been put in the oven. The boundary of the large hole goes through the center of the circular piece of dough and touches its boundary. The boundaries of the two (congruent) smaller holes touch each other, the boundary of the large hole, and the boundary of the circular piece of dough.

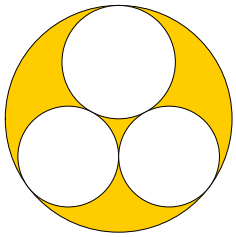

Figure 2: The remaining piece of dough is shown in yellow. The white circles represent the holes.

What is the area of the remaining piece of dough after the three circular cookies have been cut out?

Possible answers:

1. 112π cm^2
2. 115π cm^2
3. 118π cm^2
4. 120π cm^2
5. 125π cm^2
6. 128π cm^2
7. 130π cm^2
8. 132π cm^2
9. 136π cm^2
10. There is not enough information in the problem statement to compute the area of the remaining piece of dough.