



8 Cookies, Cards, and Christmas Magic

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Challenge

The elves Gwendelyn and Fredi work for Santa Claus. During their breaks, they enjoy playing cards. Every day, they bring different cookies to work since both of them like to bake. Depending on the type of cookies they have, they are more willing to share their own cookies or aim to get others' cookies.

Gwendelyn receives:

- On days 1 to 6: 20 cookies and gives away 20 cookies,
- On days 7 to 12: 40 cookies and gives away 10 cookies,
- On days 13 to 18: 30 cookies and gives away 5 cookies,
- On days 19 to 24: 50 cookies and gives away 25 cookies.

Additionally, Gwendelyn has different working hours that affect her performance in card games. Gwendelyn wins with the following probabilities:

- 50 percent on odd days divisible by three,
- 40 percent on even days,
- 20 percent on odd days not divisible by three.

Fredi receives so many cookies from Santa as a reward after the long advent season and he had so much fun with Gwendelyn that he decided to continue the idea after the Christmas season. This time, however, they are using a coin. The rule is relatively simple: a fair coin is tossed by Gwendelyn until the first "heads" appears, which ends the game. The winnings of cookies for Gwendelyn depend on the total number of coin tosses. If it's only one toss, Gwendelyn receives one cookie. With two tosses (one "tails," one "heads"), she gets 2 cookies; with three tosses, 4 cookies; with four tosses, 8 cookies, and with each additional toss, the amount of cookies doubles. Gwendelyn is excited and wonders how many cookies she expects to win this time.

The two questions are:

A) On which days during the Advent season does Gwendelyn expect to win the most cookies?

B) How many additional cookies does she expect to receive from Fredi after the Advent season?

Possible answers:

1. 22 and 0
2. 9 and 152
3. 9 and 2048
4. 9 and infinity
5. 15 and 152
6. 15 and 2048
7. 15 and infinity
8. 21 and 152
9. 21 and 2048
10. 22 and infinity