



24 Interesting Working Conditions

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Project: MATH+ Advent Calendar

Challenge

Santa Claus is a miserable employer because the number of vacation days his **nine** elves get for the next year depends on their success in a new game he invents every year. This year he has devised the following game: He puts a hat, in either red or blue, on each of his elves' heads. He does not tell them how many red and blue hats he is giving out (so it is possible that he only distributes caps in one color). Each elf can see the hat colors of all the other elves, but not their own. The common goal of the elves is that as many of them as possible find out their own hat color, because Santa promises them all together as many days off as elves "guess" their own hat color correctly.

The rules of the game are as follows: As soon as the first elf has been put on their hat, the elves are no longer allowed to talk to each other. After Santa has distributed all the hats, he calls out one elf after the other (the order is unknown to the elves beforehand). After an elf has been called, they must immediately place themselves somewhere on the red line in the storage room. As soon as one stands on the red line, one may no longer move. When the last elf has lined up, the elves write on a piece of paper the color they believe their own hat is.

After he has explained all the rules and before he distributed the hats, he gives his elves some time to work out a strategy together on how they will position themselves on the red line. Fortunately, the elves are highly logically gifted and work out an optimal strategy. We call a strategy **A** optimal if there is no strategy in which more elves write down the correct hat color with one hundred percent certainty than with strategy **A**.

Out of the nine elves, what is the largest number of elves that know certainly their hat colors with an optimal strategy?

Hint: We can assume that an elf can choose any position on the line for themselves. This means that they can squeeze between any two elves or stand next to an elf on the edge.

Possible Answers:

1. 0
2. 1
3. 2
4. 3
5. 4
6. 5
7. 6
8. 7
9. 8
10. 9