Phylogeny of elves

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Challenge

The young Christmas elves have recently learned in the Elf School about the intriguing concept of evolution and the classification of elves into a big family tree. These elves have always been known for their festive spirit and dedication to helping Santa Claus, but now they are eager to explore their own history. They were taught about how elves have evolved over time, adapting to different climates and environments.

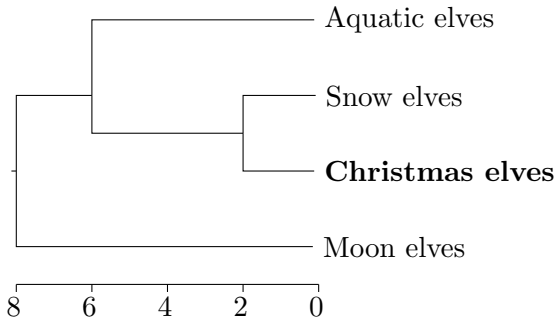

Figure 1: Phylogeny of elves. The x-axis shows the time in million years

For instance, Figure 1 displays the evolutionary history of four families of elves. Christmas and snow elves are the most closely related, sharing a common ancestor 2 million years ago. On the other hand, moon elves are the least closely related, as as a common ancestor with any other species dates back 8 million years. The times of the most recent common ancestors are collected in Table 1 and they measure the dissimilarity of the elf families.
Ten young elves wanted to learn more about their evolutionary history, so they searched for "The Great Book of Elvish Evolution" in the North Pole Archives. Unfortunately, this old book was severely damaged over the years, resulting in the loss of much of its information. However, the dissimilarity matrix was mostly intact. The only few missing entries are marked with W, X, Y, and Z in Table 2.
The little elves realized that not every value could be valid for the missing entries and it is possible to reconstruct the phylogeny from the matrix once they have the complete

	A	S	C	M
Aquatic elves (A)	0	6	6	8
Snow elves (S)	6	0	2	8
Christmas elves (C)	6	2	0	8
Moon elves (M)	8	8	8	0

Table 1: Dissimilarity matrix of elf families

	A	B	C	D	E	F	G	H	I
A	0	8	8	8	8	8	8	8	8
B	8	0	5	**Y**	**Z**	7	7	7	**W**
C	8	5	0	1	7	7	7	7	7
D	8	**Y**	1	0	7	7	7	7	7
E	8	**Z**	7	7	0	1	4	4	4
F	8	7	7	7	1	0	4	4	4
G	8	7	7	7	4	4	0	**X**	2
H	8	7	7	7	4	4	**X**	0	1
I	8	**W**	7	7	4	4	2	1	0

Table 2: Dissimilarity matrix with missing entries

matrix. After considering possibilities, each of the ten elves proposed potential values for the missing entries. However, only one of the proposals is a valid option. Please tell us which of the ten elves was correct.

Possible answers:

1. W = 7, X = 1, Y = 7, Z = 7
2. W = 8, X = 1, Y = 1, Z = 7
3. W = 7, X = 2, Y = 5, Z = 5
4. W = 7, X = 2, Y = 1, Z = 7
5. W = 8, X = 1, Y = 5, Z = 7
6. W = 8, X = 1, Y = 7, Z = 7
7. W = 7, X = 2, Y = 1, Z = 5
8. W = 7, X = 1, Y = 5, Z = 5
9. W = 7, X = 2, Y = 5, Z = 7
10. W = 8, X = 1, Y = 5, Z = 5

Project reference:

Phylogenetic trees are a mean of representing the evolutionary history of the species under study (for more details, one could check, for example, the website: http://tolweb.org/tree/). The matrix representation is an alternative way to view the relationships between the species. In a project from TU Berlin, we focus on analyzing phylogenetic trees using tropical geometry. Missing data is common in practice, so filling in the missing entries is a popular preprocessing step.