3 Revealing Sounds

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Project: Information design for Bayesian networks (MATH+ AA3-9)

Challenge

For this riddle, you slip into the role of the Grinch, who constantly gets in the way of Santa Claus and his elves. To keep you from pranks on Christmas Eve this year, the elves have made you an offer: They have been baking cookies all day, and you are supposed to guess which type was baked today. If you guess the correct type, you will receive a huge jar of fresh cookies as a gift. However, if you guess wrong, you must, in turn, promise not to play pranks on Christmas Eve. Although you are often mischievous, when it comes to such offers, you can always rely on the honesty of the elves and will therefore keep the promise yourself.

As there are only three different types of cookies (Vanilla Crescents, Nut Triangles, and Chocolate Cookies) that the elves regularly and with equal probability bake, the elves are convinced that you will guess wrong in two out of three cases, giving them good chances for a relaxed Christmas. However, you have observed the behavior of the elves very well over the past few years and have collected some information about their behavior. For example, you noticed that when baking Vanilla Crescents, the elves always listen to *Driving Home For Christmas*. On the other hand, when baking Nut Triangles, they equally likely listen to either *All I Want For Christmas Is You* or *Last Christmas*. It's different when they bake Chocolate Cookies. Although they also only listen to either *All I Want For Christmas Is You* or *Last Christmas* on a loop, in two out of three cases, *All I Want For Christmas Is You* is playing.

Which statement about the three Christmas songs and cookie types is correct?

Possible answers:

1. The song *Driving Home For Christmas* is played most frequently.

2. If you know which song was played today, you can increase your average probability of winning to more than 70%.

3. There is no song for which you can be entirely certain which cookies were baked today.

4. If *All I Want For Christmas Is You* was played, you have the highest chances of winning if you bet on Nut Triangles.

5. Nut Triangles are baked more frequently than Chocolate Cookies and Vanilla Crescents.

6. If *Last Christmas* was played, you cannot rule out any type of cookie with certainty.

7. The probability that Chocolate Cookies were baked and the elves heard *All I Want For Christmas Is You* is $\frac{2}{3}$.

8. If the song *Driving Home For Christmas* was played, each type of cookie is equally likely.

9. The probability that you heard *Last Christmas* today is 20%.

10. *All I Want For Christmas Is You* is played most frequently, and *Driving Home For Christmas* is played least frequently.

Project reference:

In the Math+ project AA3-9, the *Information design for Bayesian networks* is being investigated to determine to what extent traffic equilibria can be improved through the transmission of information. It is assumed that, based on the provided information, traffic participants can draw conclusions about the actual traffic. This occurs according to a similar principle to how the Grinch in this task draws conclusions about the baked cookie type by listening to the music.