

7 Gift Cube

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Challenge

Santa's elves are supposed to put 27 cube-shaped gifts in the warehouse. Some of the gifts are wrapped in green, the others in red. Santa wants his warehouse to look neat, but also pretty. Therefore, he gives the Christmas elves the following task:

“Dear elves, please put the 27 gifts together to form a big gift cube of size $3 \times 3 \times 3$. On each of the six sides of the cube, you have to create a pattern with exactly one green and two red gifts in each row and each column! This will be the prettiest warehouse we've ever had...”

The elves ponder how to construct the gift cube. After a while, one elf remarks, “We have too many green presents to build the big cube like the one Santa asked for!”

Another elf has an idea, “If we change the pattern so that there are exactly *two* green presents in each row and each column, then we can build the big cube with the 27 presents we have.” See Figure 1.

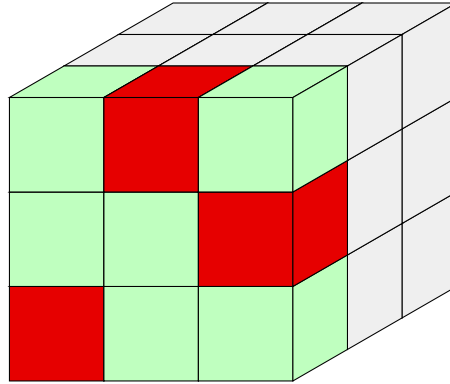


Figure 1: The big cube of 27 small square gifts and a possible admissible pattern of green and red gifts for the front side.

What is the smallest possible and the largest possible number of green gifts in the warehouse of the Christmas elves?

Possible answers:

1. The smallest possible number of green gifts is 10; the largest possible number is 13.
2. The smallest possible number of green gifts is 10; the largest possible number is 15.
3. The smallest possible number of green gifts is 12; the largest possible number is 15.
4. The smallest possible number of green gifts is 12; the largest possible number is 17.
5. The smallest possible number of green gifts is 14; the largest possible number is 17.
6. The smallest possible number of green gifts is 14, the largest possible number is 19.
7. The smallest possible number of green gifts is 16; the largest possible number is 19.
8. The smallest possible number of green gifts is 16; the largest possible number is 21.
9. The smallest possible number of green gifts is 18; the largest possible number is 21.
10. The smallest possible number of green gifts is 18; the largest possible number is 23.