8 The Canal

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Challenge

Santa's little helper Ruprecht was given an important task this year. He is responsible for supplying the cities A, B, and C with presents. Fig. 1 shows the arrangement of the cities, which all lie on a straight line heading east. The cities A and B are 5 kilometers apart, the cities B and C even 15 kilometers.

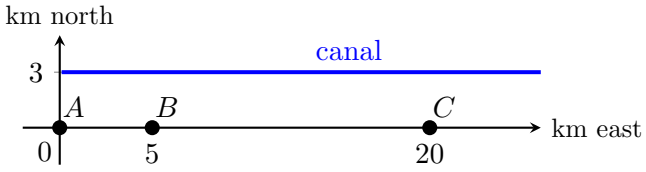

Figure 1: Arrangement of the cities and the frozen canal.

Ruprecht just finished supplying city A with presents and wants to impress Santa by delivering the cities B and C as quickly as possible. Unfortunately, there was heavy snowfall last night, and the streets connecting the cities are buried under a thick layer of snow. Therefore, Ruprecht's progress is much slower than before. However, from earlier visits to this region, Ruprecht remembers that 3 kilometers north of A there is a canal that runs straight east. Fig. 1 also shows this canal. Because of the freezing temperatures, the canal is frozen right now, and Ruprecht can ice-skate on the surface of the canal.

When Ruprecht trudges through the thick layer of snow, he manages to cover 5 kilometers per hour. On the frozen canal, with the help of his skates, he proceeds at 25 kilometers per hour.

We have the following questions:

(a) Does Ruprecht use the frozen canal on the *fastest* way from A to B?

(b) Does Ruprecht use the frozen canal on the *fastest* way from B to C?

(c) Does Ruprecht need more than 1 hour for the *fastest* way from A to B?

(d) Does Ruprecht need less than 1.8 hours for the *fastest* way from B to C?

Possible answers:

1. (a) no, (b) no, (c) no, (d) no.
2. (a) no, (b) no, (c) no, (d) yes.
3. (a) no, (b) no, (c) yes, (d) no.
4. (a) no, (b) no, (c) yes, (d) yes.
5. (a) no, (b) yes, (c) no, (d) no.
6. (a) no, (b) yes, (c) no, (d) yes.
7. (a) no, (b) yes, (c) yes, (d) no.
8. (a) no, (b) yes, (c) yes, (d) yes.
9. (a) yes, (b) yes, (c) no, (d) no.
10. (a) yes, (b) yes, (c) no, (d) yes.

Project reference:

A branch of mathematical optimisation is concerned with *optimal control problems*. Such control problems arise in a variety of real-world applications, ranging from economics over robotics to the control of the power grid of a whole country. In many cases, it can be observed that the optimal control requires a detour to reduce costs. This phenomenon is called *turnpike phenomenon*. The name is reminiscent of an observation from everyday life: when driving a long distance, it is almost always quicker to take a detour via a freeway (turnpike) than to drive slowly on the country road all the time.