11 Chocolate Appetite at Night

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Challenge

Selma, the elf girl, has a beautiful Advent calendar filled with delicious sweets. On a hook rail, hang packages labelled with the numbers 1 to 24 *in random order*. Each morning, Selma removes its well-padded contents from the package marked with the respective number. Then, she seals it properly and puts it back on its hook.

During the night from 10 to 11 December, however, it happens: Selma wakes up with a ravenous appetite for chocolate and sneaks through the sleeping house to her Advent calendar. *Randomly*, she takes off four packages in the dark and takes them back to her room—apparently unnoticed. Because the packages contain only small almost weightless items and because of the darkness, Selma cannot tell whether she is picking empty or filled packages. After raiding the packages, Selma neatly closes and returns them to the four empty hooks in the dark. In doing so, she does *not* pay attention to which package she puts back on which hook.

(a) What is the probability that Selma robbed only empty packages?

(b) What is the probability that the four numbers of the packages are consecutive numbers?

Selma's mother has heard something at night and becomes suspicious. When she takes a critical look at the Advent calendar, she might notice that not all the packages are on the same hook as they were the day before. If by chance all the packages hang in the right place, the mother does not notice anything and Selma has nothing to worry about. If there are exactly k packages in the wrong place, Selma's mother thinks it is only fair that Selma has to return $k \cdot 20\,\%$ of her Santa St. Nick's Day's candy.

(c) What is the probability that exactly three packages are hanging on the wrong hook?

(d) On average, how many packages are back in their old place after such an operation (i. e. what is the *expectation value* of the number of packages hung back correctly)?

(e) On average, what fraction of her St. Nick's Day's candy does Selma have to return?

Possible answers:

1.	(a) 0.02	(b) 1/506	(c) 1/3
2.	(a) 0.02	(b) 0.002	(d) 5/4
3.	(a) 120/253	(b) 0.002	(e) 1
4.	(a) 5/253	(c) 1/3	(d) 1
5.	(a) 5/253	(c) 0	(e) 3/5
6.	(a) 120/253	(d) 1	(e) 60 %
7.	(b) 0.002	(c) 1/2	(d) 2
8.	(b) 0.002	(c) 1/3	(e) 1
9.	(b) 1/253	(d) 2	(e) 3/5
10.	(c) 0	(d) 5/4	(e) 1