2 Arrowmatics

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Challenge

It is a public secret that all elves in Santa's crew are fond of archery, but that they dislike mathematics. In order to increase their motivation for mathematics, Santa Claus has invited the three elves Archy, Bowy, and Curvy for a combination of archery and mathematics.

Here is the general idea: Santa has installed a circular target. The elves get five arrows each. One after the other, the elves must use their bows to shoot all of their five arrows at the target. The target consists of a circular bull's eye and three concentric rings around it, as indicated in Figure 1.

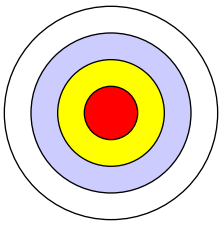

Figure 1: The circular target.

The bull's eye and every ring have each an associated fixed score value for each arrow that hits the associated target area. The elves know that these score values have been chosen from the infinite set of values

$$\{5, 10, 15, 20, 25, 30, \ldots\}$$

So each score value is a positive multiple of 5 and can be arbitrarily large. The exact score values are only known to Santa, but not to the elves. However, the elves do know the following: starting from the outer ring, the score values strictly increase when moving inside to the next ring, with the bull's eye having the highest score value of all four target areas.

After an elf has shot all five arrows at the target, Santa Claus will reveal the total score, without disclosing the scores of the individual arrows. Santa Claus challenges the elves to obtain as much information as possible on the score value of the bull's eye by carefully shooting at the target. For example, if one of the elves would be able to shoot all five arrows directly at the bull's eye, then from the total score revealed by Santa the elves could immediately deduce the exact individual score value of the bull's eye. Unfortunately, the elves are not that skilled, although all three of them manage to hit the bull's eye at least once.

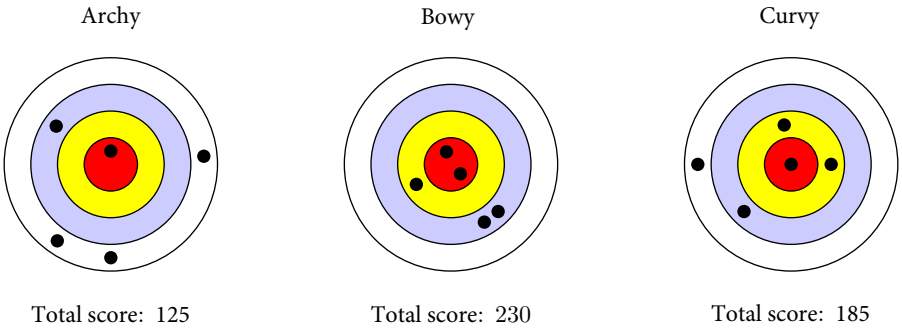

Figure 2: The spots where the arrows of Archy, Bowy, and Curvy have hit the target, and the corresponding total scores.

In Figure 2, the black dots indicate the spots where the arrows of each of the three elves have hit the target. The first target shows that Archy has hit the outer ring with three arrows, the next ring with one arrow, and the bull's eye with one arrow. Santa Claus reveals that Archy's total score is 125. From the second target, it becomes clear that Bowy is a better archer, with a total score of 230. Finally, after Curvy's five arrows have hit the target as shown on the third target, Santa reveals a total score of 185 for Curvy.

Unfortunately, with these three total scores it is not possible for the elves to determine the exact score value of the bull's eye. However, the above information limits the number of possible score values considerably.

Which of the following statements is correct?

Possible answers:

1. The number of possible score values of the bull's eye is limited to exactly 2.
2. The number of possible score values of the bull's eye is limited to exactly 3.
3. The number of possible score values of the bull's eye is limited to exactly 4.
4. The number of possible score values of the bull's eye is limited to exactly 5.
5. The number of possible score values of the bull's eye is limited to exactly 6.
6. The number of possible score values of the bull's eye is limited to exactly 7.
7. The number of possible score values of the bull's eye is limited to exactly 8.
8. The number of possible score values of the bull's eye is limited to exactly 9.
9. The number of possible score values of the bull's eye is limited to exactly 10.
10. The number of possible score values of the bull's eye is limited to exactly 11.