



23 Traces in the Snow

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Challenge

As every morning, the little elves Juri and Sergei set off to school. First, they climb the Fairy Hill, then they walk along the Pixie Stream in the Elf Valley to the enchanted fir forest. Behind the fir forest, there is an extensive snow field the two have to cross.

When Juri and Sergei leave the fir forest, they cannot believe their eyes: in the snowfield, there are traces that look like a giant coordinate system in which, as far as one can see, the lines connecting the points (m^2, m) and $(n^2, -n)$ are plotted for all $m, n \in \mathbb{N}$ with $m, n > 1$. Have aliens been at work here?

Excited, Juri and Sergei run the rest of the way to school. There, they are met by their math teacher Emma. Immediately, they tell her about their mysterious discovery. Quite interested, Emma grabs her whole class, and together they trudge to the snow field. The little elves are keen to follow on this excursion, because one day before Christmas they can hardly sit still on their classroom chairs...

“However, we still need to do some math,” Emma thinks, and asks her class which numbers on the x -axis are “hit” by these connecting lines. But be cautious: in the midst of all that excitement, one little elf was not paying attention and made a mistake. Which one?



Artwork: Julia Nurit Schönagel

Possible answers:

1. Anh answers, “I think that 1.008.988.999 is hit.”
2. Basima bets, “The number 2.517.849.199 is hit.”
3. Charly checks their notes and says, “Also 3.497.348.737 is hit.”
4. Dao deduces, “Certainly, 4.142.454.642 is hit.”
5. Elif explains, “Naturally, 5.761.648.489 is hit.”
6. Frieda figures, “The number 6.556.849.301 is hit as well.”
7. Ghazal guarantees, “Most certainly, 7.481.075.262 is hit.”
8. Hasina has the following idea, “I think 8.991.101.800 is hit.”
9. Imani is perfectly sure that, “9.768.956.129 is hit.”
10. João joins the discussion and says, “The number 10.567.846.453 is definitely hit.”