11 Christmas Bauble Mashup

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Challenge

This year, elf Willi is operating the Christmas bauble machine in the Christmas workshop. The machine produces packs of coloured Christmas baubles. You can set the package size and the possible bauble colours. However, the machine randomly selects one of the selected colours for each bauble that goes into a package. Thus, it can happen, for example, that only baubles of one colour end up in a package, although more than one colour was set. At the moment, the machine is set to 5-packs with the colours red and silver. After Willi has opened three packs, he claims, "All 5-packs with colors red and silver are different from each other." Here we call two packs *the same* if they contain the same number of balls per colour, and *different* if they are not the same.
Elf Selma replies, "That can't be true, since there are only six different packs of this kind."

(a) How many different 10-packs of red, silver, and green baubles are there?

(b) Now, the machine is set to 4-packs with the colours red and silver. What is the probability that there are exactly 3 red balls in a pack?

(c) The machine is still set to 4-packs with the colours red and silver. What is the probability that two packages produced directly one after the other are the same?

(d) Now, the machine is set to 9-packs with the colors red and silver. What is the probability that two packages produced directly one after the other are the same?

To answer the questions in parts (b) to (d), the smart Selma visualizes the filling process for two possible colours, e. g. red and silver, using a path in a coordinate system. If a silver bauble enters the pack, the path goes one step to the right; if a red bauble enters the pack, it goes one step up. In the figure below, the blue path represents the filling of a 4-pack with three silver baubles and one red bauble. The red path represents filling a 4-pack with two silver baubles and two red baubles. Maybe this illustration will also help you?
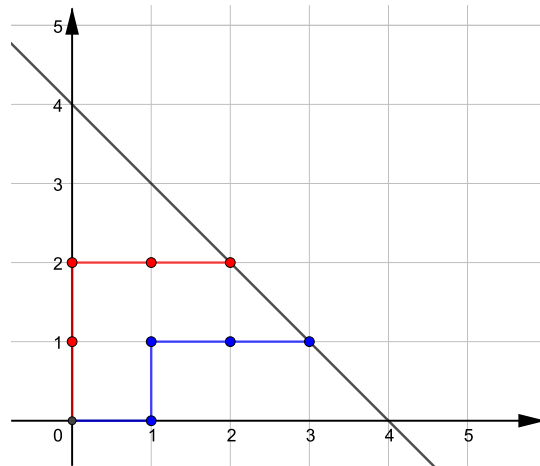
Artwork: Julia Nurit Schönnagel

Possible answers:

1. (a) 59049,   (c) $\frac{35}{128}$.

2. (a) 11,   (d) $\frac{1}{10}$.

3. (b) $\frac{1}{5}$,   (c) $\frac{1}{5}$.

4. (b) $\frac{1}{4}$,   (d) $\frac{1}{10}$.

5. (c) $\frac{35}{128}$,   (d) $\frac{12055}{65536}$.

6. (a) 66,   (c) $\frac{1}{5}$.

7. (a) 59049,   (d) $\frac{12155}{65536}$.

8. (b) $\frac{1}{4}$,   (c) $\frac{35}{128}$.

9. (b) $\frac{1}{5}$,   (d) $\frac{12155}{65536}$.

10. (c) $\frac{1}{5}$,   (d) $\frac{12055}{65536}$.