



22 Zelda in Distress

Authors: Luise Fehlinger (HU Berlin)

Florian Matthies (HU Berlin, Herder-Gymnasium Berlin)

Project: Research group “Mathematics and Education” (HU Berlin)

Challenge

Princess Zelda has been kidnapped again. This time, it is clear that courage, strength, and wisdom, which form the corners of the magic Triforce, will not be enough to save her. The fourth power, *patience*, is also crucial. Thus, the Triforce must be transformed into a magical tetrahedron. A new dimension has to be added. The equilateral triangle of the Triforce becomes a regular tetrahedron. As with the Triforce, the corners of the tetrahedron must be made up of smaller tetrahedra. These touch at their vertices, forming the scaffold of the magical tetrahedron (see Fig. 1). However, the interior must not remain empty. It must be perfectly filled by an inner solid K_1 , which in turn consists of corners and another inner solid K_2 and so on.

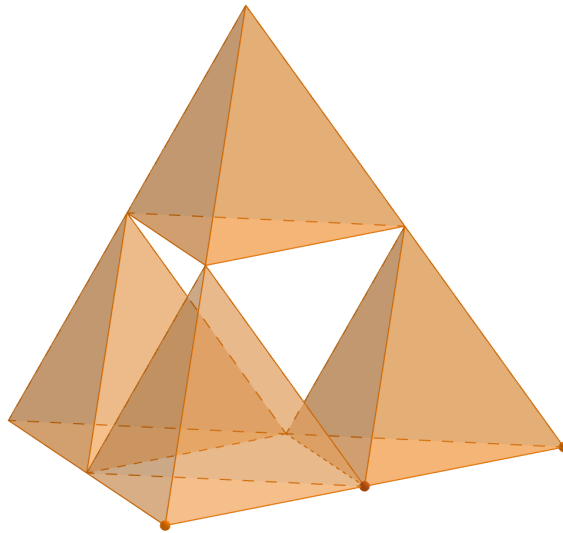


Figure 1: The tetrahedron K_0 . The solid K_1 is obtained from K_0 by cutting of the corners (in orange). Hence, K_1 is the (colourless) remainder in the center of the tetrahedron K_0 .

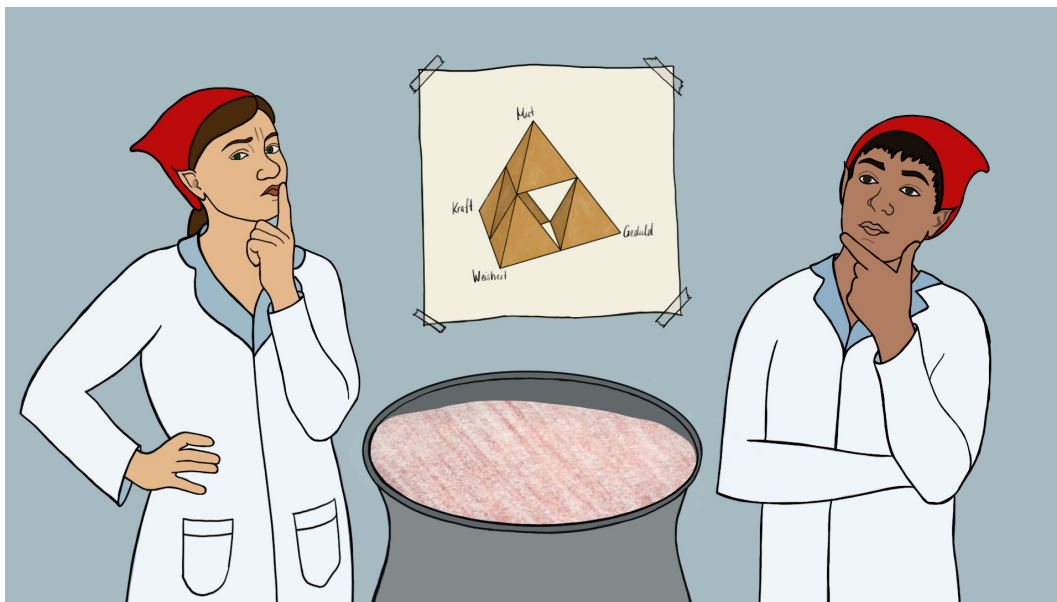
Hence, the Tetraforce is a regular tetrahedron that will be called K_0 .

- K_1 is obtained from K_0 by cutting off the corners of K_0 . The sectional surfaces are determined by the midpoints of the edges meeting in the respective vertex (see Fig. 1).
- K_2 is obtained from K_1 by cutting off the corners of K_1 . Again, the sectional surfaces are determined by the midpoints of the edges meeting in the respective vertex.
- And so on: K_{n+1} is obtained from K_n by cutting off the corners of K_n . The sectional surfaces are determined by the midpoints of the edges meeting in the respective vertex.

After an infinite number of cuts, the magical heart of the Tetraforce remains.

This heart will be manufactured by the elves—who else would be able to do this—from a magic crystal, which must be cast from Christmas elixir at -40°C at the magnetic pole of the Earth.

But what are the properties of the magical heart? The elves make various guesses. But one statement is wrong. Which one?



Artwork: Frauke Jansen

Possible answers:

1. K_1 is an octahedron.
2. The volume of K_1 is only half of the volume of K_0 .
3. All of the K_n are convex.
4. K_0 is the only one of the solids where only three edges meet in each vertex. For all other K_n , four edges meet in each vertex.
5. Each of the K_n has faces that are equilateral triangles.
6. The faces of the K_n are triangles and/or quadrilaterals.
7. The centroids of the faces of K_n are part of all K_{n+k} ($k \in \mathbb{N}$).
8. In each step, the number of edges is doubled.
9. Each solid K_n besides K_0 has two faces more than it has vertices.
10. The magical heart is a ball.