16   Reindeer Election

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Challenge

Once and for all, Santa has had enough of the quarrals among the five chief reindeer, Rudolph, Blitzen, Comet, Dasher, and Dancer, about who gets to lead Santa's sleigh on Christmas Eve this year. Therefore, he has decided to let the entire population of Elf Valley vote.

The election will have four rounds of voting: in the first round, Reindeer 1 competes against Reindeer 2. The winning reindeer from the first round will compete against reindeer 3 in the second round. In turn, the winning reindeer from this round competes against reindeer 4 in the third round. The reindeer that won the third round competes against reindeer 5 in the fourth (and final) round.

A round of voting is won by the reindeer that receives more than 50 % of the votes. The reindeer that wins the last is the winner of the election and will lead Santa's sleigh on Christmas Eve.

Santa is very pleased with his brilliant idea. Now, he just has to figure out how to determine the order of reindeer 1 through 5. Since he is quite sure that this will not affect the outcome of the election anyway, and Rudolph is secretly his favorite reindeer, Rudolph gets to decide the order.

Very pleased with Santa's choice, Rudolph takes a look at the predicted voting behavior of the people of Elf Valley—but quickly reckons that his chance of winning the election is pretty low. In fact, you can quite acurately tell how the population of Elf Valley will vote. According to this, the population is divided exactly into five equally sized groups with the following preferences:

Group 1: Dasher > Dancer > Blitzen > Rudolph > Comet

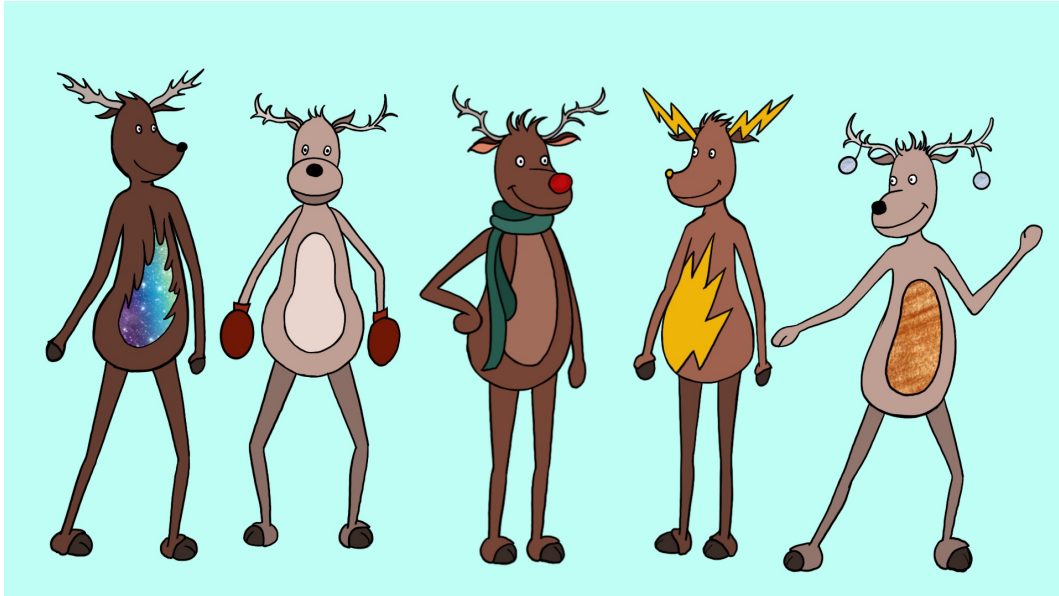**Group 2:** Dasher > Dancer > Rudolph > Comet > Blitzen

Group 3: Comet > Blitzen > Dasher > Dancer > Rudolph

Group 4: Dancer > Comet > Blitzen > Dasher > Rudolph

Group 5: Dancer > Blitzen > Dasher > Rudolph > Comet

We also know that the population of Elf Valley is very reliable: on election day, all residents of Elf Valley exercise their right to vote and vote exactly as predicted.

Which order of the reindeer should Rudolph choose to be allowed to lead the sleigh as the winner of the election process?

Artwork: Frauke Jansen

Possible answers:

1. Comet, Dasher, Dancer, Blitzen, Rudolph.

2. Dasher, Dancer, Blitzen, Rudolph, Comet.

3. Dasher, Dancer, Rudolph, Comet, Blitzen.

4. Dasher, Blitzen, Dancer, Comet, Rudolph.

5. Dasher, Blitzen, Dancer, Rudolph, Comet.

6. Dancer, Blitzen, Dasher, Comet, Rudolph.

7. Dancer, Dasher, Comet, Rudolph, Blitzen.

8. Dancer, Dasher, Blitzen, Comet, Rudolph.

9. Dancer, Dasher, Comet, Blitzen, Rudolph.

10. There is no order for which Rudolph is able to win the election.